Worksheet 3 Karnaugh maps **Answers**

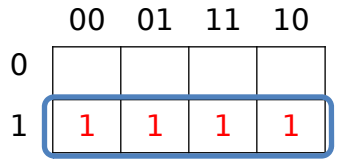
Task 1

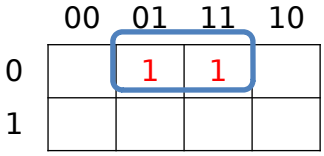
1. Fill in Karnaugh maps for the following expressions, showing the groupings by ringing them.

   In parts (d), (e) and (f), fill in the missing row and column labels.

   (a) A (b) ¬A ˄ C

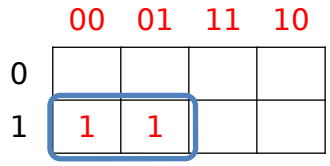
BC A

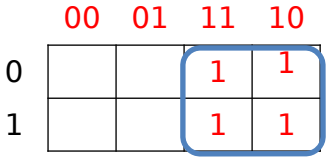


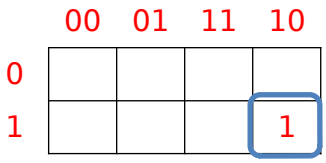


(c) A ˄ ¬B (d) B (e) A ˄ B ˄ ¬ C

BC A







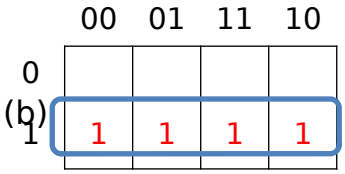
2. Fill in Karnaugh maps for the following expressions, showing the groupings. Hence simplify the expressions.

   (a) (A ˄ B) ˅ (A ˄ ¬C) ˅ (A ˄ ¬B) = A

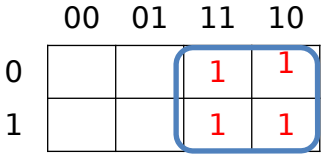
   (b) (A ˄ B ˄ C) ˅ (¬A ˄ B) ˅ (A ˄ B ˄ ¬C) = B

BC A

(a)



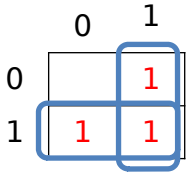
(b)



3. Use a Karnaugh map to show that A ˅ ¬A ˄ B = A ˅ B.

   (You drew a truth table to prove this in Worksheet 2, Question 4)

B A



Task 2

4. What Boolean expressions do each of the ringed squares in the Karnaugh map in Figure 1 represent?

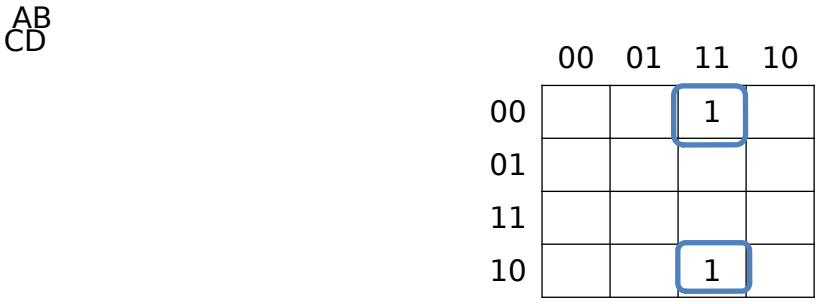


Figure 1

Row 1 column 3 = (¬A ^¬ B ^ C ^ D)

Row 4 column 3 = (A ^ ¬B ^ C ^ D)

Write the Boolean expression represented by the map in its simplest form.

The two ringed squares form one group which "wraps around" to represent

¬B ^ C ^ D

5. (a) Ring the two groups in Figure 2. What Boolean expression does this Karnaugh map represent? (¬B ^ C ^ D) ˅ (A ^ B ^ D)

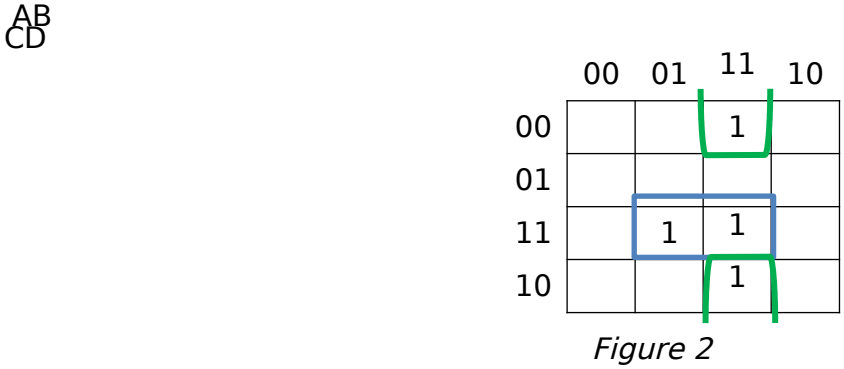


Figure 2

(b) Complete the Karnaugh map in Figure 3 to represent the expression:

(A ∧ B ∧ C ∧ ¬D) ∨ (¬A ∧ C ∧ ¬D) ∨ (A ∧ ¬B ∧ C ∧ ¬D)

Draw the resulting group(s) and hence simplify the expression.

Simplified expression: C ∧ ¬ D

AB
CD

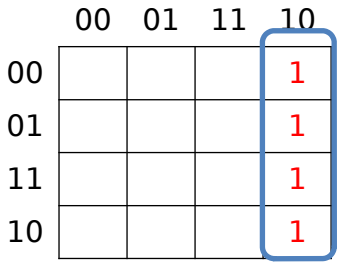


Figure 3

(c) Complete the Karnaugh map in Figure 4 to represent the expression:

(A ∧ B ∧ C) ∨ (C ∧ D) ∨ (A ∧ ¬C) ∨ (A ∧ ¬B ∧ C ∧ ¬D)

Draw the groups, and hence simplify the expression.

Simplified expression: A ∨ (C ∧ D)

AB
CD

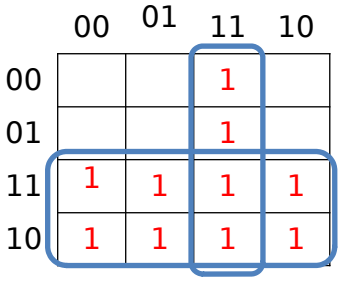


Figure 4

(d) Complete the Karnaugh map in Figure 5 to represent the expression:

$(\neg A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (\neg A \wedge \neg B \wedge C \wedge \neg D) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D)$

$\vee (A \wedge \neg B \wedge C \wedge \neg D)$

Draw the group(s), and hence simplify the expression.

Simplified expression: $\neg B \wedge \neg D$

Note: When the corners are filled in, they all wrap around to make one group similar to a folded piece of paper.

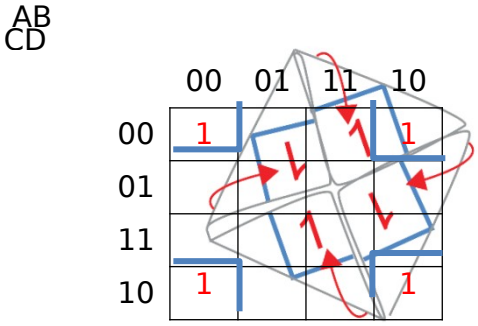


Figure 5

(e) Complete the Karnaugh map in Figure 6 to represent the expression:

$(\neg A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (\neg A \wedge \neg B \wedge C \wedge \neg D) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D)$

$\vee (A \wedge \neg B \wedge C \wedge \neg D) \vee (\neg B \wedge D)$

Draw the group(s), and hence simplify the expression.

Simplified expression: $\neg B$

The top and bottom rows wrap round to form a single group

AB
CD

	00	01	11	10
00	1	1	1	1
01				
11				
10	1	1	1	1

Figure 6

7. How many squares in a Karnaugh map with 4 variables contain 1 when an expression containing only AND symbols has:

(i) 4 variables, e.g. $(\neg A \wedge \neg B \wedge \neg C \wedge \neg D)$? 1

(ii) 3 variables e.g. $(A \wedge B \wedge C)$? 2

(iii) 2 variables, e.g. $(\neg B \wedge D)$? 4

(iv) 1 variable, e.g. A? 8